AD-A118 100 ARMY INST OF DENTAL RESEARCH WASHINGTON DC F/G 6/5
LINEAR PROGRAMMING: A RESOURCE ALLOCATION METHODOLOGY FOR DENTA--ETC(U)
JUN 82 J D SHULMAN
UNCLASSIFIED NL

1 of 1
AD A
118100

END
DATE
FILMED
9 82
DTIC

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A118100	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Linear Programming: A Resource Allocation Methodology for Dental Managers		5. TYPE OF REPORT & PERIOD COVERED Submission of paper
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Jay D. Shulman, LTC, DC		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Institute of Dental Research Walter Reed Army Medical Center Washington, DC 20012		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Medical Research and Development Command HQDA-IS Fort Detrick, MD 21701		12. REPORT DATE June 1982
		13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Mathematical programming, linear programming, dental management, management science, resource allocation, dental public health, health administration.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The use of mathematical programming in health care has increased markedly in the past decade. This paper addresses the use of one form of mathematical programming, linear programming, for resource allocation, and provides an overview of the technique through the use of graphical examples relating to dentistry. The linear programming model is described and its interpretation is illustrated through the use of a sensitivity analysis. The role of the dental program managers as consumers of linear programs is discussed.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

82 08 11 007

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

LINEAR PROGRAMMING: A RESOURCE ALLOCATION

METHODOLOGY FOR DENTAL MANAGERS

Jay D. Shulman, DMD, MA, MSPH

Lieutenant Colonel, U.S. Army Dental Corps

10512 Kinloch Rd.
Silver Spring MD 20903

The author is currently Assistant Deputy for Research at the United States Army Institute of Dental Research, Walter Reed Army Center, Washington, DC 20012.

Abstract

The use of mathematical programming in health care has increased markedly in the past decade. This paper addresses the use of one form of mathematical programming, linear programming, for resource allocation, and provides an overview of the technique through the use of graphical examples relating to dentistry. The linear programming model is described and its interpretation is illustrated through the use of a sensitivity analysis. The role of the dental program managers as consumers of linear programs is discussed.

Introduction.

As the level of governmental funding of the health sector declines competition for scarce health care dollars will increase. Such competition is hardly new to dental public health; but in the past decade decision-makers have begun to rely heavily on quantitative decision techniques from the fields of operations research and management science in making resource allocation decisions. The past twenty years have seen a steady increase in the number and sophistication of these techniques used in health care. Program evaluation and review technique (PERT), cost-benefit analysis, cost effectiveness analysis, and mathematical programming are frequently used to describe, analyze, and solve problems in the health sector.[1] The purpose of this paper is to provide an overview of one mathematical programming technique, linear programming. While specific methods of solving linear programs are beyond the scope of this paper, it is important that dental program managers have an understanding of linear programming and the kinds of problems for which it is suited.

Discussion.

Mathematical programming is best understood in the context of resource allocation. Managers have at their disposal a limited supply of resources such as personnel, equipment, materiel, and money. They must decide how to use, or allocate these resources in such a way as to get the greatest return.[2] A resource allocation problem has three common elements: an

objective, a range of possible solutions for reaching that objective, and constraints (limitations) on the solution, or decision. The manager seeks to find the best, or optimal, solution that satisfies the constraints.[3]

Consider the problem of determining how many dentists and how many dental assistants to hire for a new dental clinic. This staffing decision lends itself to mathematical programming, a technique that "allows the implicit evaluation of all alternatives and the identification of the 'best' or optimal one... ."[4] Expressed in mathematical programming terms the problem is: "what is the optimal mix of dental assistants and dentists?" In this example, the function that we are trying to make optimal, or optimize, is the amount of patient treatment. It could just as easily be the amount of profit, or any other management objective. Optimizing can take the form of minimizing or maximizing. For the purposes of this illustration the objective will be to find the number of dentists (X) and assistants (Y) that would maximize the units of care provided (Z).[5]

To do this it is necessary to determine the productivity of an average dentist with different numbers of assistants. In a clinic with multiple operatories, for example, assume that in a dentist with one assistant provides 1000 units of care; a dentist with two assistants produces 1300 units; and a dentist with three assistants provides 1600 units.[6] In effect, the productivity contribution of a dentist (DENT) is 700 units and that of each

assistant (ASST) is 300 units. The values of DENT and ASST are the parameters of the objective function of the problem.

The problem is expressed by the equation

$$\text{Max } (Z) = A\ X + B\ Y;$$

where A and B are the parameters DENT and ASST, respectively, and X and Y are variables representing the optimum number of dentists and assistants. The objective function, or equation to be maximized in this problem is

$$\text{Max } (Z) = 700\ X + 300\ Y.$$

Note that in a mathematical programming problem the parameters must be known while the variables must be solved for.

Consider the limitations, or constraints on the solution. One likely constraint is the total number of personnel, for example, a ceiling of 15. This may be due to space limitations (e.g., a limited number of operatories), or a limitation on hiring. Examples of further constraints are dentist-assistant proportions (the dentist-assistant proportion may not be greater than 1:3); efficiency (every dentist must have at least 1 assistant); the number of dentists (a minimum of 2); and a salary budget ceiling ($250,000). Assume that the dentists 'cost' $50,000 while assistants 'cost' $20,000. These constraints can be expressed as inequalities:

PERS:	X	$+$	$Y \leq 15$		[1]
MIX1:	$3X$	$-$	$Y \geq 0$		[2]
MIX2:	X	$-$	$Y \leq 0$		[3]
MINDENT:	X		≥ 2		[4]
BUD:	$50{,}000X$	$+$	$20{,}000Y \leq 250{,}000$		[5]

NON-NEGATIVITY: $X \geq 0$ [6]

NON-NEGATIVITY: $Y > 0$ [7]

LINEARITY: There is a linear relationship between X and Y. Equations 6 and 7 are redundant since there must be at least 2 dentists (constraint 4) and at never fewer assistants than dentists (constraint 3). While in this problem X and Y cannot be zero, equations 6 and 7 are included because we are interested in only positive values for X and Y since a negative number of dentists and assistants would be meaningless. The non-negativity assumption is emphasized because it is written into almost all linear programming computer codes.

Linearity assumes that the objective function and the constraints are linear. In the objective function it was assumed that X and Y substitute for each other in a linear fashion; that the productivity contribution of dentists is 700 units while that of assistants is 300 units for any combination of dentists and assistants. For example, in a clinic with 2 dentists, the first assistant provides the same productivity contribution as the 10th assistant (300 units). It is the assumption of linearity that makes this a linear programming problem.

At this point the the series of inequalities can be solved. The solution of linear programming problems can be quite complex. While manual techniques exist, the availability of computer codes makes manual solution unnecessary. To demonstrate what these codes and techniques actually do, however, let us examine a graphical solution of the problem.[7]

To solve for X and Y, the constraint inequalities are plotted. The personnel constraint [1] is plotted in Figure 1. Since it is a 'less than' constraint the area of interest, or feasible region is bounded by the X and Y axes (since X and Y are non-negative) and is contained in triangle abc.

[Insert Figure 1 about here]

In Figure 2 the first dentist-assistant proportion constraint [2] is added. Since it is a 'greater than' inequality the area of interest is on and to the right of the line, narrowing the feasible region to triangle bcd.

[Insert Figure 2 about here]

In Fig. 3 the second dentist-assistant proportion constraint [3] is added. The feasible region is now contained in triangle cde.

[Insert Figure 3 about here]

In Figure 4 the minimum number of dentists constraint [4] is added. It is a 'greater than' constraint and the area of interest is on and to the right of the line, narrowing the feasible region to degf.

[Insert Figure 4 about here]

The budget constraint [5], is added in Fig. 5. The feasible region is now in gfhi.

[Insert Figure 5 about here]

Figure 6 shows the feasible region and all possible integer solutions for X and Y that satisfy the constraints.[8]

[Insert Figure 6 about here]

After the feasible region is determined, Z, the objective function, is plotted. To do this it is necessary to arbitrarily assign a value to Z. Setting Z at 4,900 (units), for example, the equation (or isovalue line) is plotted in the same way the inequalities were plotted (Fig. 7). Since this line does not enter the feasible region, a smaller Z is used and the process is repeated until the isovalue line enters the feasible region. This process generates a family of lines with the same slope. The first integer coordinates in the feasible region that the isovalue line passes through are located at point x. The solution to the problem, then, lies at point x (X=3, Y=5).

[Insert Figure 7 about here]

The maximum number of dentists and assistants that may be hired is 3 and 5, respectively, and they would provide 3,600 units of care per week.

The solution satisfies all the constraints and maximixes productivity. There are fewer than 15 people [1], there is at least 1 dentist for each 3 assistants [2], there is at least 1 assistant for each dentist [3], there are at least 2 dentists [4], and the salary expenditure is less than $250,000 [5]. The

next best solution (X=3, Y=4, Fig. 6) also satisfies all the constraints but provides only 3,300 units of care a week.

If the problem were a minimization problem; finding the number of dentists and assistants that would minimize productivity given the same constraints, the solution would be at point y (X=2, Y=2). The two dentists and two assistants would provide 2,000 units of care per week.

Sensitivity Analysis

After the solution is found it is useful to examine the sensitivity of the solution to changes in the constraints. Would, for example, a ten thousand dollar budget increase (to $260,000) change the solution? To find this out the revised constraint is graphed. Figure 8 shows the revised feasible region that results when the budget constraint line (hi) is replaced with the new constraint (h'i').

[Insert Figure 8 about here]

Note that the feasible region has grown to fgi'h' and includes another set of integer solutions at point y (X=3, Y=6). Line a is the isovalue line that provided the solution to the problem in Figure 7. Isovalue line b passes through point y, the new solution to the problem. Increasing the budget by $10,000, then, changes the solution, allowing an additional assistant to be hired, resulting in the production of 300 additional units of care per week.

Another sensitivity that is worth looking at is the sensitivity of the solution to changes in the parameters of the objective function, the productivity contributions of dentists and assistants. To do this a new isovalue line is plotted for each alternative set of parameters. Line a (Figure 9) is the isovalue line that provides the solution for the problem when A=600 and B=400.

[Insert Figure 9 about here]

The solution is still at point x (X=3, Y=5). Line b is the isovalue line for A=800 and B=200. The solution is unchanged although the slopes of lines a and b are different. This shows that in this particular instance, the solution (the optimal staffing mix) is relatively insensitive to changes in projected relative production contribution of dentists and assistants. Thus, although a manager might not be certain about the accuracy of A and B he could still be relatively confident in the solution.

Role of the Dentist-Manager.

It is hoped that 'walking through' the solutions to these simple but somewhat strained problems has provided the reader with a feel for the linear programming approach to problem solving. This is necessary because there are potential problems associated with applying a linear programming solution if the technique is not properly understood.

The first problem is that the model can be sensitive to inaccurately specified parameters. While this was shown not to be a problem in Figure 9, solutions often change quite radically with minor changes in parameters. Managers should be confident of the validity of the parameter estimates, and even then they should perform a sensitivity analysis on them. It is hardly necessary to regraph the problem to realize that changing the dentists' and assistants' salaries in the budget constraint to $60,000 and $15,000, respectively, will yield a different solution.

The second problem area lies in omitting significant constraints or imposing unrealistic or trivial constraints on the solution. The addition of unnecessary constraints may decrease the size of the feasible region reducing the number of options available to the manager that will satisfy all the constraints. Mutually exclusive constraints permit no feasible region, and consequently no solution.[9] Conversely, the omission of important constraints in the model may result in a feasible region that contains a mathematically valid solution that cannot be realistically applied, because of the existence of that constraint in the real world.

Conclusion.

Dental program managers are not expected to solve linear programming problems; that is the job of operations researchers and management scientists. They should, however, be sufficiently familiar with the technique to understand the product with which

they are being provided. They should be capable of asking intelligent questions, and recognizing flawed answers. Most of all they should not be awed by reams of computer printout and consultants who act as if they have just returned from Olympus. The role of the dental program manager is that of a consumer of the technology. As an informed consumer the public health dentist will work more effectively with management scientists; be a better manager and a more effective competitor for resources in his organization.

Endnotes

1. Brant Fries, "Bibliography of Operations Research in Health-Care Systems: An Update." Operations Research, 27 (March-April, 1979), p. 408.

2. Robert J. Thierauf, Decision Making Through Operations Research. (New York: John Wiley & Sons, 1975), p. 158.

3. William L. Dowling, "The Application of Linear Programming to Decision-Making in Hospitals." Hospital Administration 16 (Summer, 1971), p. 66.

4. D. Michael Warner & Don C. Holloway, Decision Making and Control for Health Administration. (Ann Arbor, Michigan: Health Administration Press, 1978), p. 188. While the number of alternatives in this problem is not large, mathematical programming is capable of dealing with problems where the number of alternatives is very large (even infinite).

5. For the purpose if this example procedure units are weighted by dollar value.

6. This is an oversimplification. The marginal productivity rate will decrease with each additional assistant. This results in a non-linear relationship between X and Y and would require a more complicated (non-linear programming) approach.

7. This example is composed of two decision variables and five constraints and can be solved graphically in two dimensions. Problems involving more than two decision variables cannot be solved in this manner and are nearly always solved by computer.

8. Since dentists and dental assistants are integer quantities, non-integer solutions (e.g., fractions of dentists and assistants) are generally undesirable. Such a solution might be of value to a manager if he were interested in a mix of part time and full time personnel. Often when the variables can take only integer values, a solution procedure is used that yields integer solutions only. This technique is complex and beyond the scope of this discussion. For the purposes of these examples only integer solutions will be considered.

9. A relatively new technique, goal programming, allows the analyst to prioritize the constraints. The solution process attempts to satisfy the constraints in decreasing order of importance. Thus, a problem for which linear programming would yield no feasible solution (no feasible region) goal programming would yield a solution even though some constraints are violated. See, for example, Sang M. Lee, "An Aggregate Resource Allocation

Model for Hospital Administration", *Socio-Economic Planning Science*, 7 (1973) 381-395.

Figure 1

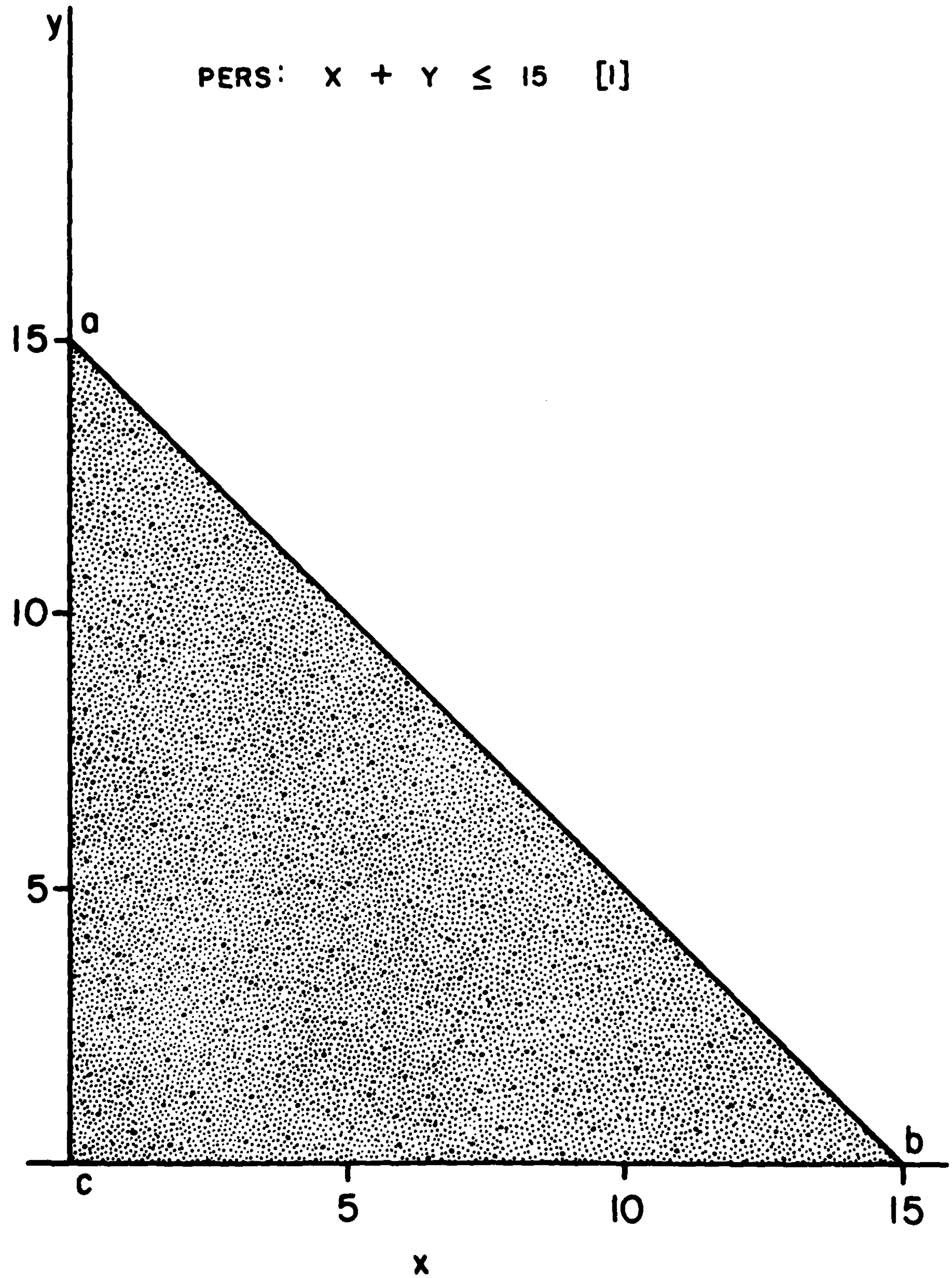


Figure 2

PERS: $X + Y \leq 15$ [1]

MIX 1: $3X - Y \geq 0$ [2]

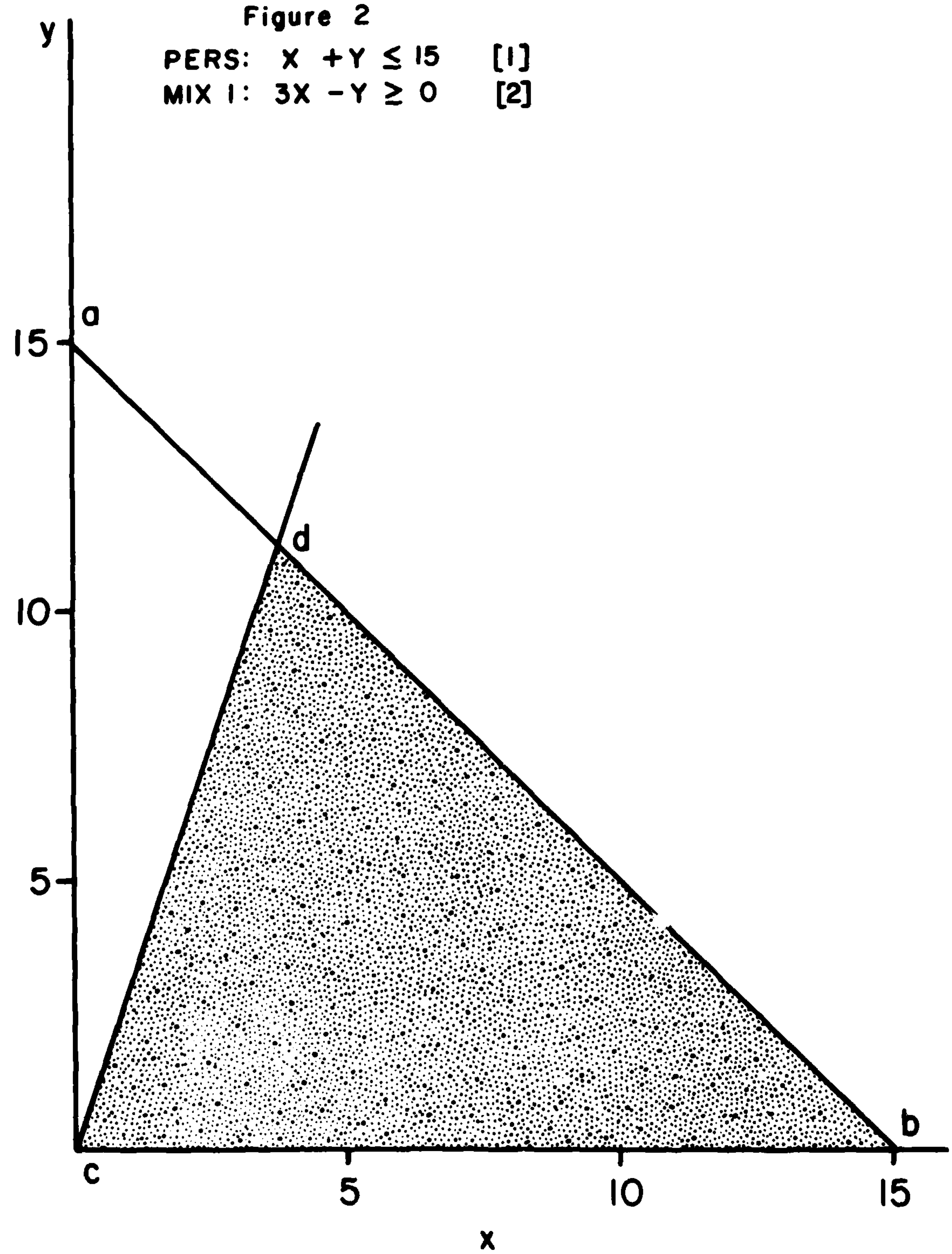


Figure 3

PERS: $X + Y \leq 15$ [1]

MIX 1: $3X - Y \geq 0$ [2]

MIX 2: $X - Y \leq 0$ [3]

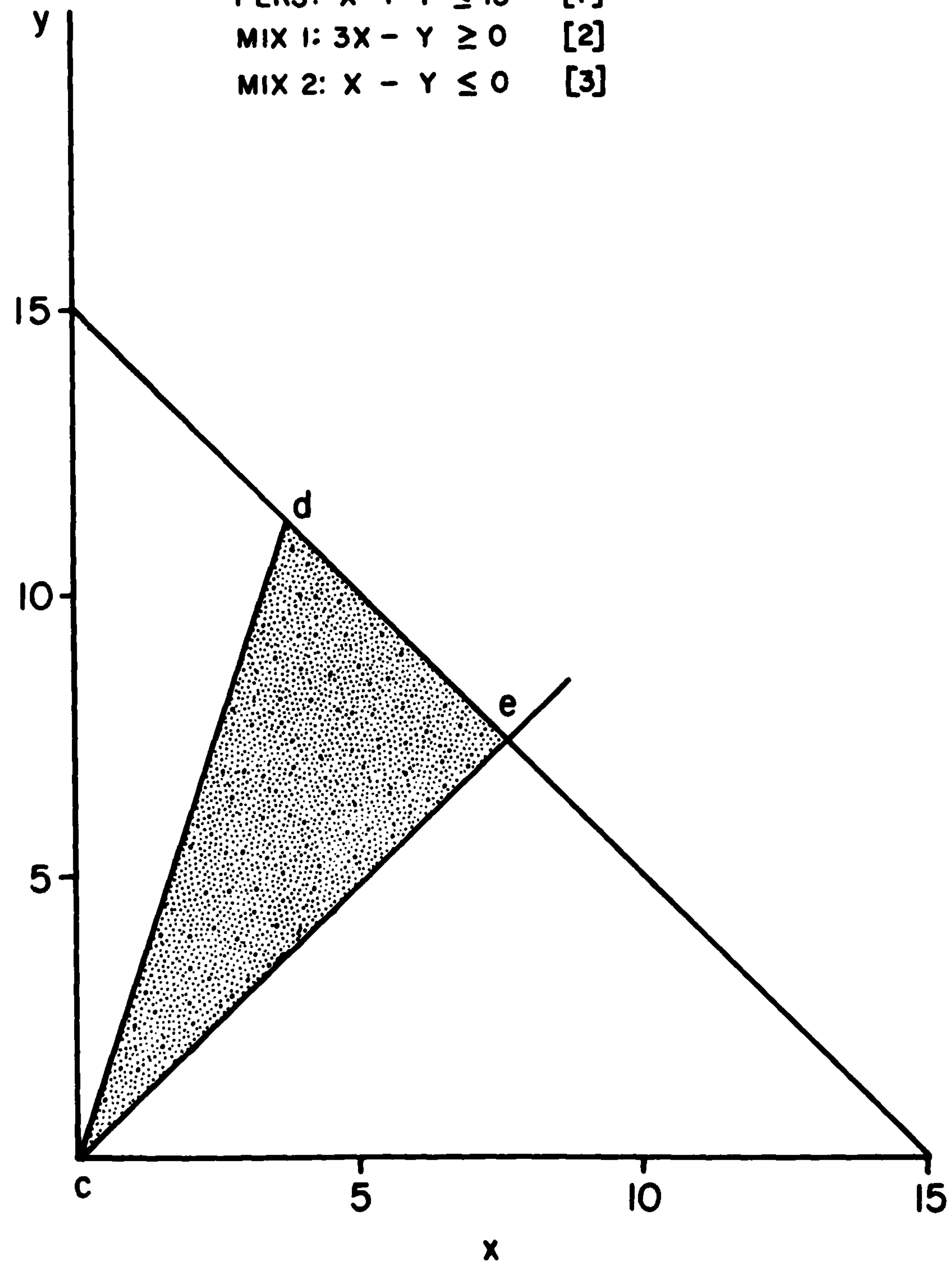


Figure 4

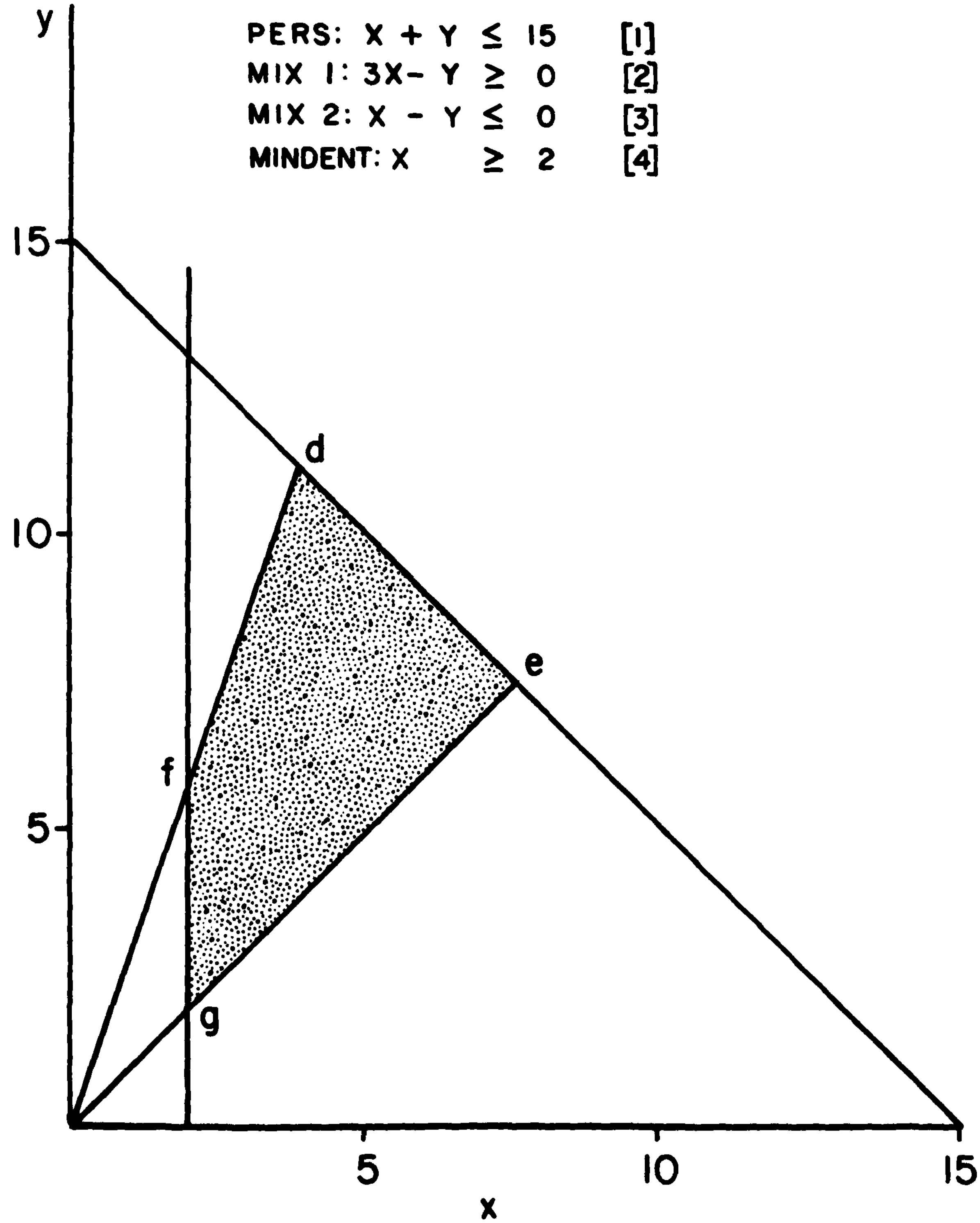
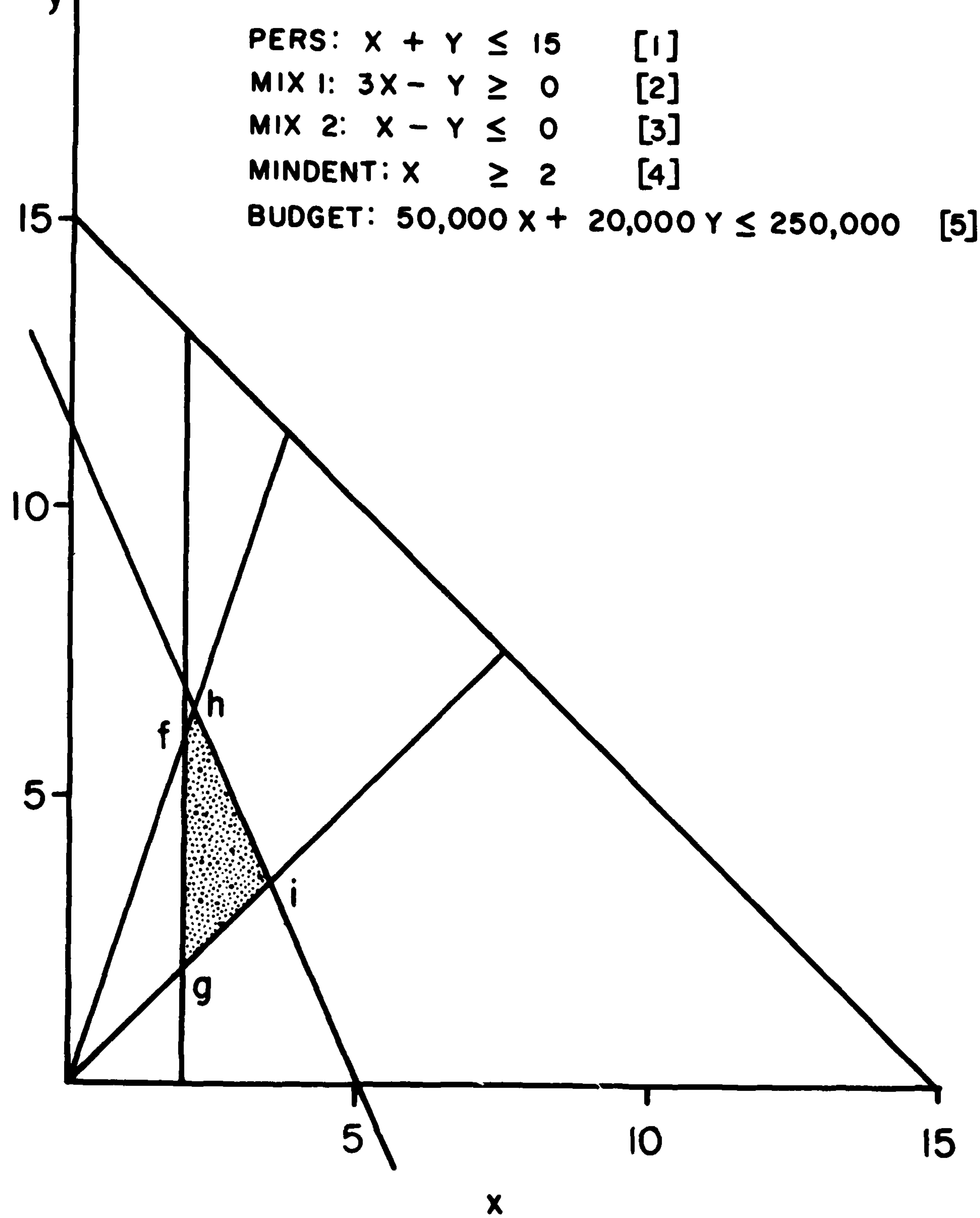

PERS: X + Y ≤ 15 [1]
MIX 1: 3X− Y ≥ 0 [2]
MIX 2: X − Y ≤ 0 [3]
MINDENT: X ≥ 2 [4]
y
15
10
5
d
e
f
g
5
10
15
x

Figure 5



PERS: X + Y ≤ 15 [1]
MIX 1: 3X − Y ≥ 0 [2]
MIX 2: X − Y ≤ 0 [3]
MINDENT: X ≥ 2 [4]
BUDGET: 50,000 X + 20,000 Y ≤ 250,000 [5]
y
15
10
5
x
5
10
15
f
h
i
g

Figure 6

FEASIBLE REGION WITH INTEGER SOLUTIONS

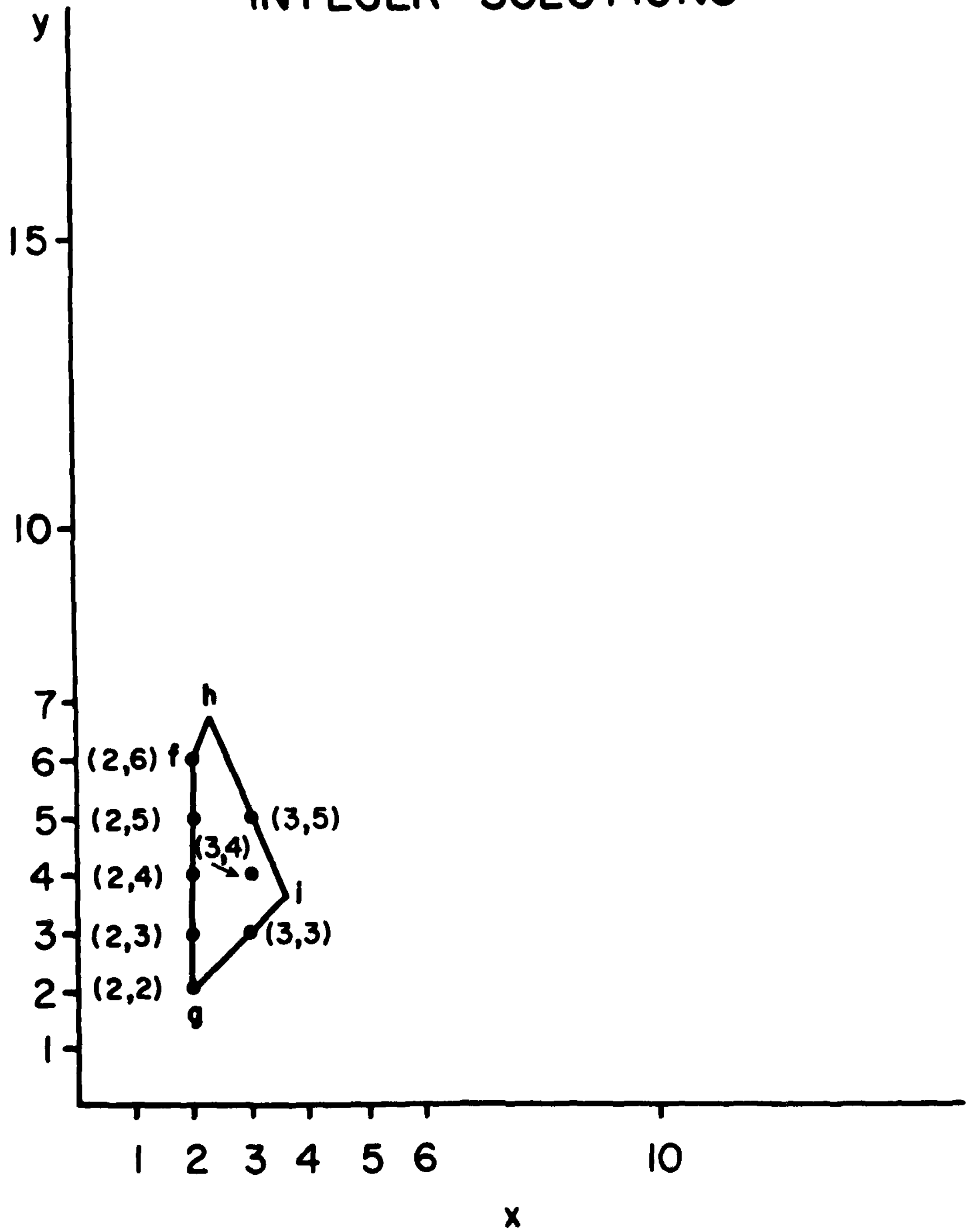
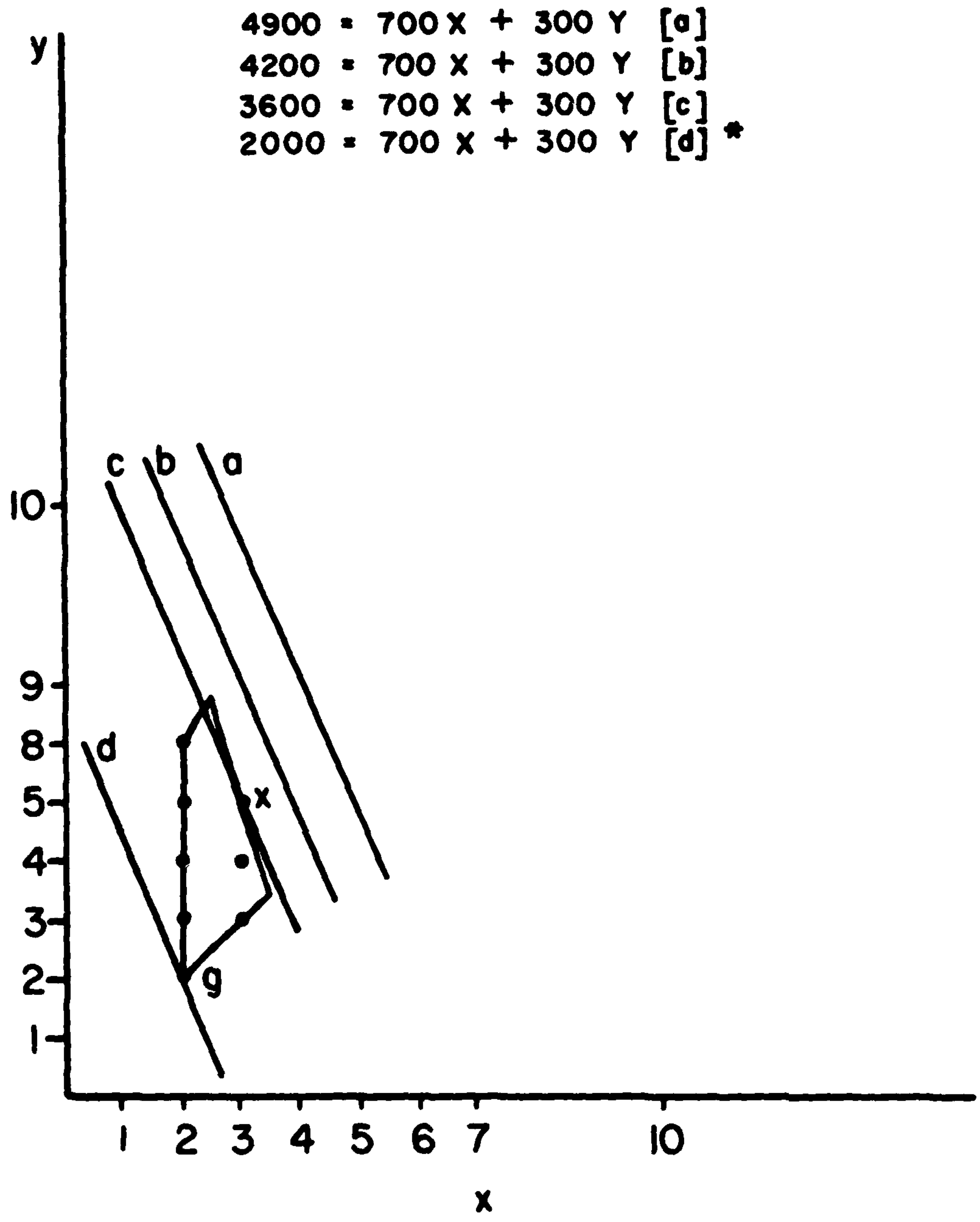


Figure 7

FITTING ISOVALUE LINES BY TRIAL AND ERROR



* MINIMUM

Figure 8

SENSITIVITY ANALYSIS OF CHANGED BUDGET CONSTRAINT

BUD: $50{,}000X + 20{,}000Y \leq 250{,}000$

BUD': $50{,}000X + 20{,}000Y \leq 260{,}000$

ISOVALUE LINES:

$3600 = 700X + 300Y$ [a]
$3900 = 700X + 300Y$ [b]

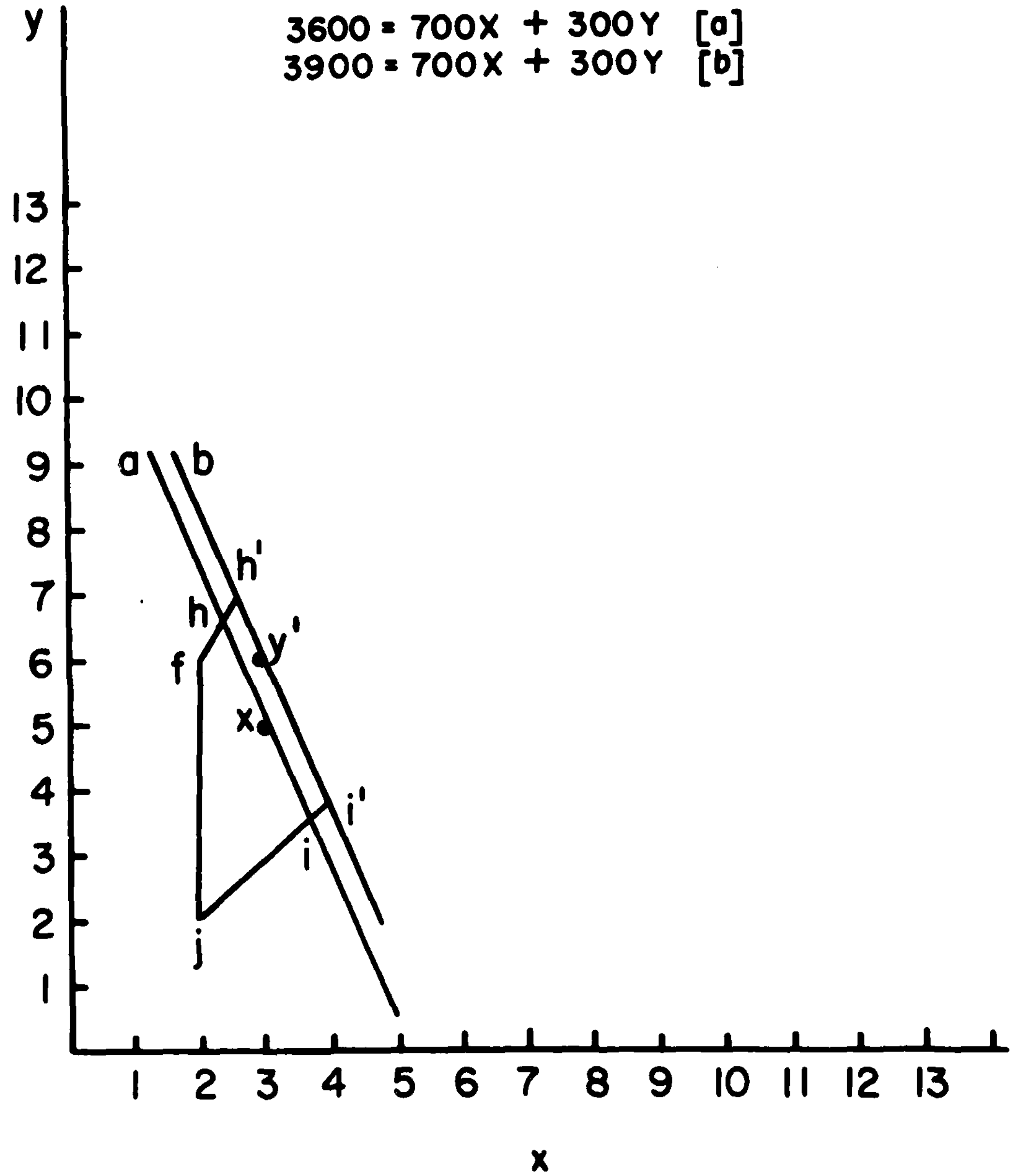
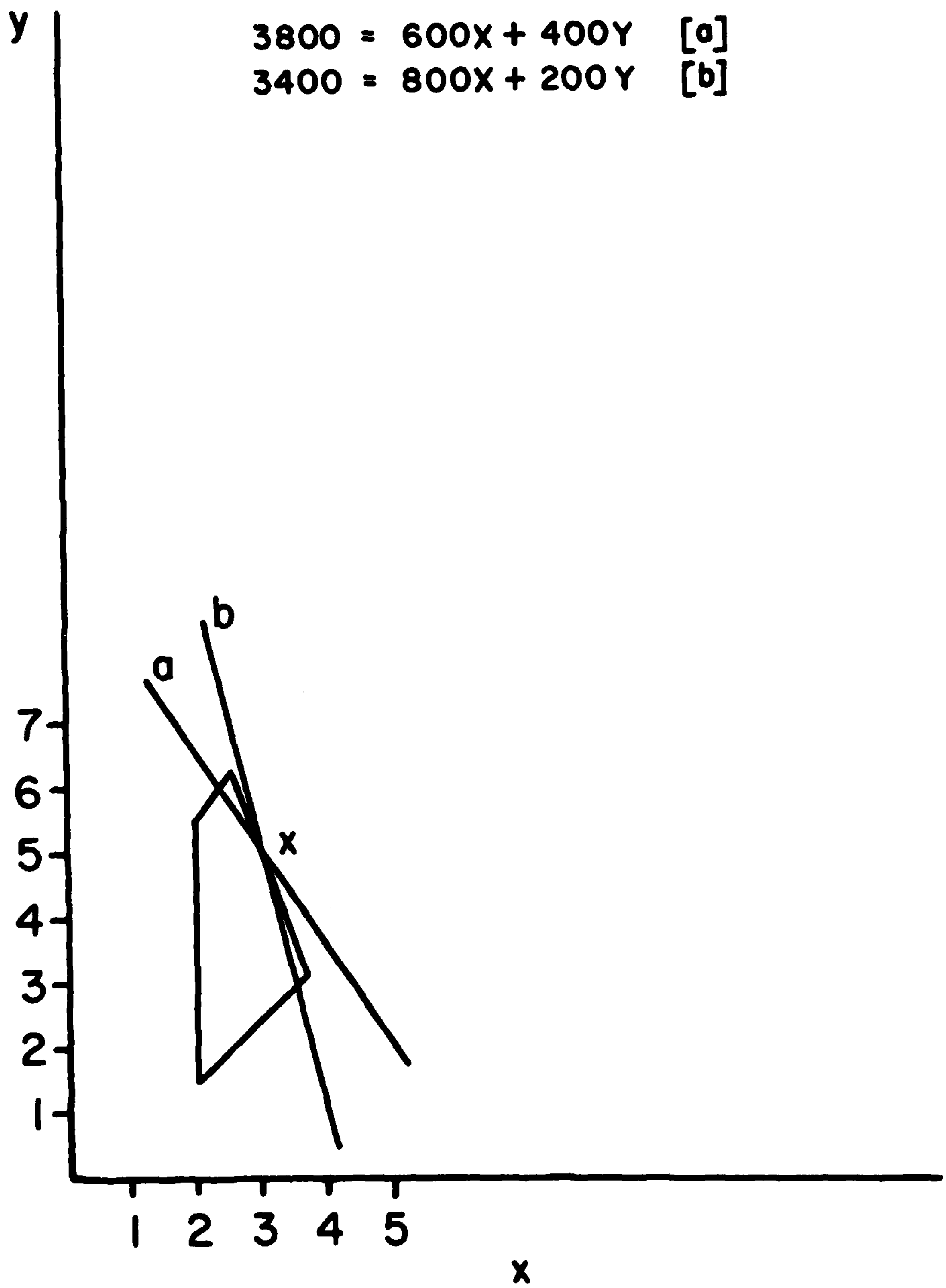


Figure 9

ISOVALUE LINE

$3800 = 600X + 400Y$ [a]
$3400 = 800X + 200Y$ [b]



ATE

LMED

8